ABOUT TRANSFINITE COMPOSITIONS OF WEAK EQUIVALENCES OF HIGHER DIMENSIONAL TRANSITION SYSTEMS

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ABSTRACT. This note will be never published. In two published papers [Gau11] [Gau14], it is implicitly assumed that the classes of weak equivalences of the model structures constructed are closed under transfinite composition because they are finitely accessible and accessibly embedded. It turns out that the argument which is given can only prove that they are accessible and accessibly embedded. In this note, this strong argument is replaced by a weaker one which is easy to check.

1. THE CORRECT ARGUMENT

It is about the model structures constructed and studied in [Gau11] and in [Gau14] on the categories of weak and cubical transition systems $\mathcal{WTS}$ and $\mathcal{CTS}$.

All generating cofibrations of $\mathcal{WTS}$, $\mathcal{CTS}$, $\mathcal{CTS}^+$ are maps between weak transition systems containing a finite number of states and actions by [Gau11, Proposition 5.4] for $\mathcal{WTS}$, by [Gau14, Theorem 4.6] for $\mathcal{CTS}$ and by definition of $\mathcal{CTS}^+$ by [Gau14, p 116]. Hence by [RR15, Proposition 4.1], the classes of weak equivalences of all these model structures as well as of all left Bousfield localizations are closed under transfinite compositions.

2. CORRECTION

This fact is used in the proofs of [Gau11, Theorem 7.10], [Gau11, Proposition 8.5], [Gau14, Proposition 7.14] and [Gau14, Proposition 8.5].

3. CONJECTURES

The following conjectures sound reasonable.

3.1. **Conjecture.** *The model categories of* [Gau11] *and* [Gau14] *are* $\aleph_0$-*combinatorial.*

The difficulty for proving Conjecture 3.1 is to find explicit sets of generating trivial cofibrations for all these model structures. We only know that such sets exist using Olschok's methods.

3.2. **Conjecture.** *The classes of weak equivalences of the model categories of* [Gau11] *and* [Gau14] *are finitely accessible.*

References

[Gau11] P. Gaucher. Towards a homotopy theory of higher dimensional transition systems. *Theory Appl. Categ.*, 25:No. 25, 295–341 (electronic), 2011. 1

[Gau14] P. Gaucher. Homotopy theory of labelled symmetric precubical sets. *New York J. Math.*, 20:93–131 (electronic), 2014. 1

[RR15] G. Raptis and J. Rosický. The accessibility rank of weak equivalences. *Theory Appl. Categ.*, 30:No. 19, 687–703 (electronic), 2015. 1

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