

M2 MPRI ECOLO: Exploring Computational Models through LINEAR LOGIC

3 lecturers			
Claudia Faggian	17/09-15/10	CNRS researcher at IRIF (4 th floor, SG)	faggian@irif.fr (*) emails: pls always MPRI in subject !
Delia Kesner	22/10, 5/11, 12/11, 19/11, 10/12		
Giulio Manzonetto			

How to *communicate*?

- Email (see above)
- To facilitate collective discussion, we have set a (rather experimental) **discord server**: for invitation, just email me "MPRI discord"



INSTITUT
DE RECHERCHE
EN INFORMATIQUE
FONDAMENTALE

1

Plan /topics (for my lessons)

HANDS-ON

Linear Logic, its syntax & dynamics

Focus on 2 key aspects of LL:

- a *dynamic, interactive view on computation*, centered on **how the computation is performed**, rather than the input-output behavior
- takes into account the amount of **resources** needed to produce an output, yielding to *quantitative approaches to types and semantics*

- Theoretical tools to study the operational properties of a system:
 - Rewrite Theory (rewriting=abstract form of program execution)
- Bridging between lambda-calculus and functional programming:
 - Call-by-Value and Call-by-Name, weak and lazy calculi.
- Openings on current research (LL is an active research area!):
 - probabilistic programming and Bayesian learning
 - Bayesian proof-nets (and quantum ones)

(Internships possible on proof-theoretical & operational aspects of

➤ **Bayesian** (probabilistic) inference/programs

➤ **Quantum** inference/programs

➤ **Tensor networks**

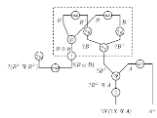
2

Linear Logic [Girard87] breakthroughs

New insights into **proof theory** and
(via the **Curry-Howard correspondence between proofs and programs**)
into the **theory of programming language and in semantics**.

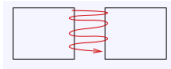
Powerful tools to study the process of
computation

- **Proof Nets**: advanced graph formalism



- representation of proofs (λ-terms, functional programs) by graphs
- Powerful tools for the analysis of cut-elimination (= execution) as graph-rewriting process

• Dynamic view, capturing the **flow of computation**:



➤ **Game Semantics**

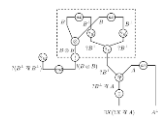
➤ **Geometry of Interaction**

3

Linear Logic [Girard87] breakthroughs

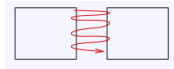
New insights into **proof** (via the **Curry-Howard correspondence**)
into the **semantics of programs** (via the **Curry-Howard correspondence**)

• **Proof Nets**: advanced formal system



- representation of proofs (λ-terms, functional programs) by graphs
- tool for the analysis of cut-elimination (= execution) as graph-rewriting process

• Dynamic view, capturing the **flow of computation**:



➤ **Game Semantics**

➤ **Geometry of Interaction**

• **Account for resources**

- **Quantitative Semantics**
- **Quantitative Type Systems**

4

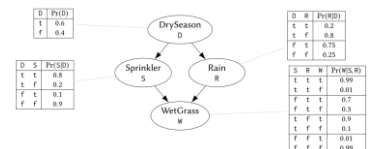
Resource awareness (Quantitative Types)

$\frac{\mathcal{P} \vdash x : P}{\mathcal{P} \vdash x : P} \text{ S-VAR}$	$\frac{\mathcal{P} \vdash b : B}{\mathcal{P} \vdash b : B} \text{ S-BOOL}$	$\frac{\mathcal{P} \vdash u : P \quad \mathcal{P}, x : P \vdash t : A}{\mathcal{P} \vdash \text{let } x = u \text{ in } t : A} \text{ S-LET}$
$\frac{\mathcal{P} \vdash v : L_1 \quad \mathcal{P} \vdash w : L_2}{\mathcal{P} \vdash (v, w) : L_1 \otimes L_2} \text{ S-PAIR}$	$\frac{\mathcal{P} \vdash v : L_1 \otimes L_2 \quad \mathcal{P}, x : L_1, y : L_2 \vdash t : A}{\mathcal{P} \vdash \text{letp } (x, y) = v \text{ in } t : A} \text{ S-LETP}$	
$\frac{\mathcal{P}, x : P \vdash t : A}{\mathcal{P} \vdash \lambda x. t : P \multimap A} \text{ S-ABS}$	$\frac{\mathcal{P} \vdash t : P \multimap A \quad \mathcal{P} \vdash v : P}{\mathcal{P} \vdash t v : A} \text{ S-APP}$	
$\frac{\mathcal{P} \vdash t : A}{\mathcal{P} \vdash !t : !A} \text{ S-BANG}$	$\frac{\mathcal{P} \vdash v : !A}{\mathcal{P} \vdash \text{der } v : A} \text{ S-DER}$	
$\frac{}{\Lambda, x : P \vdash x : P} \text{ I-VAR}$	$\frac{\Lambda, \Gamma_1 \vdash u : P \quad \Lambda, \Gamma_2, x : P \vdash t : A}{\Lambda, \Gamma_1 \cup \Gamma_2 \vdash \text{let } x = u \text{ in } t : A} \text{ I-LET}$	
$\frac{\Lambda \vdash v : L_1 \quad \Lambda \vdash w : L_2}{\Lambda \vdash (v, w) : L_1 \otimes L_2} \text{ I-PAIR}$	$\frac{\Lambda \vdash v : L_1 \otimes L_2 \quad \Lambda, x : L_1, y : L_2, \Gamma \vdash t : A}{\Lambda, \Gamma \vdash \text{letp } (x, y) = v \text{ in } t : A} \text{ I-LETP}$	
$\frac{\Lambda, \Gamma, x : P \vdash t : A}{\Lambda, \Gamma \vdash \lambda x. t : P \multimap A} \text{ I-ABS}$	$\frac{\Lambda, \Gamma_1 \vdash t : P \multimap A \quad \Lambda, \Gamma_2 \vdash v : P}{\Lambda, \Gamma_1 \cup \Gamma_2 \vdash t v : A} \text{ I-APP}$	
$\frac{\Lambda, \Gamma_1 \vdash t : A_1}{\Lambda, \Gamma_1 \cup \Gamma_2 \vdash t : A_1} \text{ I-BANG}$	$\frac{\Lambda, \Gamma \vdash v : [A]}{\Lambda, \Gamma \vdash \text{der } v : A} \text{ I-DER}$	

5

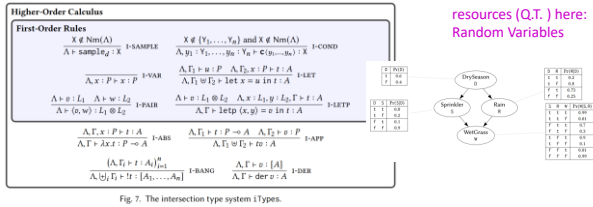
Bayesian Networks

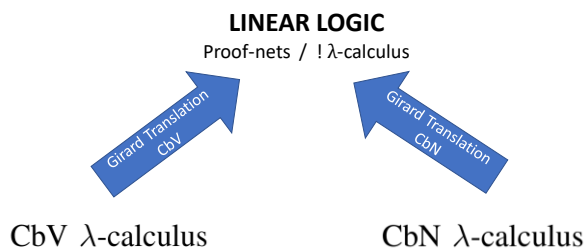
resources (Q.T.) here:
Random Variables



6

Higher-Order Bayesian Networks





13

Resources

- **Webpage:** <https://www.irif.fr/~faggian/MPRI/>
- **Lecture Notes** (by A. Middeldorp, O. Laurent)
- **Handbook of Linear Logic** (drafty, but readable)

15

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14

Practical modalities

- 5 lectures:

17/09/25, 24/09/25, 1/10/25, 08/10/25, 15/10/25

- **Grading:** ? ?

- Regular short ~~homework~~ exercises

- Regular short in-class exercises, some homework



16