Correction du devoir # 1

MAT 3541

Ex 1 (4 pts)

$$\mathcal{M}_{\mathcal{B}}(f) = \begin{array}{cc} & \begin{array}{ccc} f(e_1) & f(e_2) & f(e_3) \end{array} \\ & \begin{pmatrix} 3 & 0 & -2 \\ 0 & 5 & 7 \\ 1 & 1 & 1 \end{pmatrix} \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array} \end{array}$$

$$\det f = \det \mathcal{M}_{\mathcal{B}}(f) = 4$$

Ex 2 (3 pts)

$$\operatorname{Ker} D = \left\{ P \,/\, \frac{dP}{dx} = 0 \right\} = \{ a_0 \,/\, a_0 \in \mathbb{R} \} = \mathbb{R}$$

$$\operatorname{Im} D = \left\{ P \,/\, \exists\, Q \in \mathbb{R}[X],\ \frac{dQ}{dX} = P \right\} = \mathbb{R}[X]$$

Ex 3 (7 pts)

$\operatorname{Im}(f+g) = \{ f(x) + g(x) \,/\, x \in V \}$

et $\operatorname{Im} f + \operatorname{Im} g = \{ f(x) + g(y) \,/\, x, y \in V \}$

donc $\operatorname{Im}(f+g) \subseteq \operatorname{Im} f + \operatorname{Im} g$

donc $\operatorname{rang}(f+g) \leq \dim(\operatorname{Im} f + \operatorname{Im} g)$

Or $\dim(\operatorname{Im} f + \operatorname{Im} g) \leq \dim \operatorname{Im} f + \dim \operatorname{Im} g$

d'où $\operatorname{rang}(f+g) \leq \operatorname{rang} f + \operatorname{rang} g$

Ex 4 (6 pts)

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \operatorname{Ker} F \iff MA = 0$$

$$\iff \begin{cases} a - c = 0 \\ b - d = 0 \\ -2a + 2c = 0 \\ -2b + 2d = 0 \end{cases}$$

$$\iff \begin{cases} a = c \\ b = d \end{cases}$$

donc $\operatorname{Ker} F = \operatorname{Vect}\left(\left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \right\} \right)$ et $\dim \operatorname{Ker} F = 2$

$$A = \begin{pmatrix} x & y \\ z & t \end{pmatrix} \in \operatorname{Im} F \iff \exists\, A' = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad \Pi A' = A$$

$$\iff \exists\, a, b, c, d \quad \begin{cases} x = a - c \\ y = b - d \\ z = -2a + 2c \\ t = -2b + 2d \end{cases}$$

$$\iff \exists\, e, f \quad \begin{cases} x = e \\ y = f \\ z = -2e \\ t = -2f \end{cases}$$

<u>donc</u> $\operatorname{Im} F = \operatorname{Vect}\left(\left\{ \begin{pmatrix} 1 & 0 \\ -2 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & -2 \end{pmatrix} \right\}\right)$ <u>et</u> $\operatorname{rang} F = 2$